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During collision of π - mesons with deuterons, their scattering and also their conversion into neutron mesons can occur. The cross-sections of scattering and conversions into a neutral meson were calculated (1) with the aid of the theory of perturbations for various possible types of bond between π-mesons and nucleons. A series of data concerning the character of the interaction of mesons with nucleons, in particular the dependence upon spin, can be analyzed by comparison of the data on scattering (and on conversion into a neutral meson) in hydrogen and deuterium. Theoretical considerations here do not require any assumptions concerning the smallness of the interaction.

Let us apply the semiphenomenological method, employed earlier (2) to the problem of the scattering of fast neutrons by deuterons and to the problem of the capture of π-mesons by deuterons. Let a π-meson collide with a proton. We shall calculate the familiar amplitude of the scattered meson (charged or neutral) for a given angle of scattering. If the meson possesses the spin 0, then the amplitude of scattering must be scalar; the amplitude, however, corresponding to the flight of the neutral meson must be scalar if the internal "count" of the charged and neutral mesons are the same, and must be pseudo-scalar if they are opposite.

From this requirement we can determine the character of the possible dependence of amplitude upon the nucleon's spin. Namely:

1) amplitude of scattering:

[Note: "μ_π" should be "u_π" throughout.]

$$\mu_\pi = a + \vec{b}\cdot\vec{\sigma} \tag{1}$$

where a is a scalar (which is generally a function of the angles), sigma $\vec{\sigma}$ is the spin operator, and $\vec{b}$ is a pseudovector; here

$$\vec{b} = b_0\, \vec{k}\times\vec{k}', \tag{1a}$$

where b_0 is a scalar function of the angles, and $\vec{k}$ and $\vec{k}'$ are the initial and final impulses (momenta) of the meson.

The same structure is possessed by the expression for the amplitude of conversion of a π-meson into a neutral μ_{π^0} for the same "count" [u_{π^0}]

$$\mu_{\pi^\circ} = A + \vec{B}\vec{\sigma} \tag{1b}$$

In the case, however, of different "count" we have:

$$\mu_{\pi^0}^{(-)} = \vec{c}\vec{\sigma} \tag{2}$$

where $\vec{c}$ is a vector (also depending upon the angles).

Corresponding cross-sections for collisions with hydrogen, averaged over the orientations of the nucleon's spin, equal:

$$\begin{aligned} \sigma_\pi &= /a/^2 + /b/^2 = \sigma_a + \sigma_b \\ \sigma_{\pi^0} &= /A/^2 + /B/^2 = \sigma_A + \sigma_B \\ \sigma_{\bar{\pi}^0} &= /c/^2. \end{aligned} \tag{3}$$

Let us now consider the collision of negative a π-meson with a deuteron, which collision leads to its conversion to a neutral meson. In so far as the process proceeds only because of the interaction with the proton, the amplitude of scattering will then be

$$\mu_{\pi^o}^{D} = \int \Psi^*(\rho)\cdot e^{i(\vec{k}-\vec{k}')\vec{\rho}/2} \mu_{\pi^o D} \Psi(\vec{\rho})\,(d\vec{\rho}) \qquad (4)$$

where $\vec{\rho}/2$ is the radius-vector of the proton (in the system of the deuteron's center of inertia); $\Psi_D(\rho)$ is the wave function of the deuteron; $\Psi(\vec{\rho})$ is the wave function of the two neutrons forming as a result of the collision.

Let us derive the expression for the cross section averaged over the spin states of the nucleons and summed over the states of the neutrons (the expression is effected by the same method as in (1)):

$$\sigma_{\pi^o}^{D} = \frac{1}{2}(\sigma_A + \frac{2}{3}\sigma_B)F_+ + \frac{1}{2}(\sigma_A - \frac{1}{3}\sigma_B)F_- , \qquad (5)$$

where

$$F_{\pm} = \frac{1}{2}\int \Psi_D^*(\rho)\,(e^{i(\vec{k}-\vec{k}')\vec{\rho}/2} \pm e^{-i(\vec{k}-\vec{k}')\vec{\rho}/2})\,\Psi_D(\vec{\rho})\,(d\vec{\rho})$$

If we use for Ψ_D the following expression

$$\Psi_D = \sqrt{\frac{\alpha}{2\pi}}\cdot e^{-\alpha\rho}/\rho,$$

where $\hbar^2\alpha^2/M$ is the deuteron's energy of bond, then we have

$$\sigma_{\pi o}^{D} = \sigma_A + \sigma_B - (\sigma_A + \frac{1}{3}\sigma_B)\,\frac{2\alpha}{|\vec{k}-\vec{k}'|}\,\text{arc tan}\,\frac{|\vec{k}-\vec{k}'|}{2\alpha} \qquad (6)$$

Similarly in the case of different "counts" of mesons, we have

$$\sigma^{D^-}_{\pi^0} = \sigma^-_{\pi^0} \left(\frac{2}{3} F_+ + \frac{1}{3} F_-\right) \tag{7}$$

or

$$\sigma^{D^-}_{\pi^0} = \sigma^-_{\pi^0} \left(1 - \frac{1}{3} \frac{2\alpha}{/\vec{k}-\vec{k}'/} \text{ arc tan } \frac{/\vec{k}-\vec{k}'/}{2\alpha}\right) \tag{8}$$

We note the essential absence of cases of similar or different "count" of π and and π^o -mesons. For $\vec{k}'$ close to $\vec{k}$, then σ^D (6) tends to 0 because (1a) $\vec{B} = B_0 \vec{k} x \vec{k}'$, but we have $(\text{arc tan } x/x)_{x=o} = 1$, at the same time as $\sigma^{D^-}_{\pi^0} = 2\sigma^-_{\pi^0}/3$

If there exists a connected state of the system of two neutrons (deuterons), then its formation as a result of collision is possible. Here the system of nucleons must pass from the triplet to the singlet state in order that, in correspondence with Pauli's principle, the two neutrons might be found in the S-state.

The corresponding cross-sections are:

$$\sigma^{D\to 2n}_{\pi^0} = \frac{1}{3}\, \sigma_{\pi^0} \frac{16\alpha\beta}{/\vec{k}-\vec{k}'/^2} \left(\text{arc tan} \frac{/\vec{k}-\vec{k}'/}{2(\alpha+\beta)}\right)^2 \tag{9}$$

where σ_{π^0} is σ_B or $\sigma^{\mp}_{\pi^0}$, and $\hbar^2 \beta^2/M$ is the dineutron's energy of bond. Therefore the amplitude of scattering is:

$$\mu^D_\pi = /\Psi^*(\vec{\rho}) \left(\mu_\pi e^{i(\vec{k}-\vec{k}')\vec{\rho}/2} + \mu'_\pi e^{-i(\vec{k}-\vec{k}')\vec{\rho}/2}\right) \Psi_D(\vec{\rho}) \, (d_\gamma \vec{\rho}\ ^-) \tag{10}$$

Here μ'_π is the amplitude of scattering on neutron, and $\Psi(\vec{\rho})$ is the final state of the neutron and proton. The scattering cross-section summed over the neutron and proton's states of motion possesses the following form:

$$\sigma^D_\pi = \sigma_a + \sigma_b + \sigma'_a + \sigma'_b + \frac{4\alpha}{/\vec{k}-\vec{k}'/} \text{ arc tan } \frac{/\vec{k}-\vec{k}'/}{2\alpha} \cdot Q$$

where Q is

$$Q \equiv \cdot(\sqrt{\sigma_a \sigma'_a} \cdot \cos \delta_a + \frac{1}{3} \sqrt{\sigma_b \sigma'_b} \cdot \cos \delta_b), \qquad (11)$$

where $\sigma_a' + \sigma_b'$ are the cross-section of scattering on neutron, and δ_a is the phase difference of the quantities a and a'; similarly for δ_b.

In the work (3) devoted to a similar review of the scattering of a π-meson on deutron, the presence of two types of scattering by nucleons (the types corresponding to amplitudes a and b) was not taken into consideration.

For elastic scattering we have:

$$\sigma_\pi^{D-D} = \frac{16\alpha^2}{/\vec{k}-\vec{k}'/^2} \left(\arc \tan \frac{/\vec{k}-\vec{k}'/}{4\alpha}\right)^2 [\sigma_a + \sigma_a^- + 2\sqrt{\sigma_a \sigma'_a} \cdot \cos \delta_a +$$

$$+ \frac{2}{3}(\sigma_b + \sigma'_b + 2\sqrt{\sigma_b \sigma'_b} \cdot \cos \delta_b)] \qquad (12)$$

Literature Cited

1. V. Berestetskiy and I. Pomeranchuk. Ibidem [Doklady], Volume 77, page 803 (1951).

V. Berestetskiy and I. Shmushkevich. Zhurnal Eksper i teoret Fiziki, Volume 21, No. 12 (1951).

2. I. Pomeranchuk. Ibidem [Doklady], Volume 78, page 249 (1951); Zhurnal Eksper i teoret Fiziki, Volume 21, page 1113 (1951); ibidem [Doklady], Volume 80, pages 47 (1951).

G. F. Chew. Physical Review, Volume 80, page 196 (1950).

3. S. Fernbach, T. A. Green, and K. M. Watson. Physical Review, Volume 82, page 980 (1951).

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